# Transfer of learning between isomorphic artificial domains: Advantage for the abstract

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## Abstract

Transfer between isomorphic domains was investigated. Thirty college undergraduate students learned two isomorphic artificial systems. One system was concrete in the sense that it was perceptually rich and dynamic, while the other was abstract, involving written symbols. The results show significant positive transfer from the abstract domain to the concrete domain and no significant transfer from the concrete to the abstract.

## Introduction

One of the goals of successful learning is transfer, or the ability to apply acquired knowledge outside the learned situation. Although a desired outcome of learning, spontaneous transfer is notoriously difficult to achieve. In the past few decades, numerous studies document poor or non-existent spontaneous transfer across isomorphic situations (Ben-Zeev & Star, 2001; Gholson et al., 1997; Holyoak, Junn, & Billman, 1984; Holyoak & Koh, 1987; Schoenfeld & Herrmann, 1982). Poor performance has been attributed to surface features distracting from underlying structure.

Which aspects of the learning situation facilitate transfer? A widely held belief in the education community has been that learning and transfer of mathematical and scientific knowledge is facilitated by the use of concrete representations of more abstract mathematical and scientific principles. In the past several decades, the use of concrete representations has been a growing part of the mathematics curriculum. Concrete representations include both physical manipulatives as well as specific instantiations of abstract concepts. They are often perceptually rich and meaningful. Mathematical concepts are traditionally represented in an abstract symbolic form, while applications of the mathematics to scientific and real-world scenarios can be thought of as concrete instances of the abstract concept.

The National Council of Teachers of Mathematics (NCTM) reform movement launched in 1989 promoted the role of such representations in the curriculum. For example, Dienes blocks (Dienes, 1960) are used in elementary mathematics education to teach arithmetic and place value. Dienes blocks are concrete proportional representations of the base-ten number system. The belief of educators who use the blocks is that through their use, young children will not only be able to represent and execute arithmetic problems, but will also be able to gain insight into the structure of the base-ten number system (Fuson & Briars, 1990).

Much support for the use of either concrete manipulatives or concretely situated applications in the learning of mathematics comes from constructivist educators. Cobb, Yackel, and Wood (1992) propose that students actively construct mathematical knowledge in social contexts. Furthermore, they suggest that topics which are applications of mathematics such as those from real-world or scientific situations provide good initial instructional activities. That is, instruction of mathematical concepts should be initiated through applications of the mathematics as opposed to initiated in symbolic form.

These approaches to learning and transfer seem to echo the Piagetian theory, according to which education should parallel the process of cognitive development, and the ability for abstraction is not achieved before the formal operational stage (Inhelder & Piaget, 1958). At the same time, children's reasoning during the preceding stage (i.e., the concrete operational stage) was said to be limited to objects and physically possible situations. If learning parallels the process of development, then transfer from more concrete to more abstract representations should be more efficient than the reverse.

However, there are strong reasons to doubt this view. First, it has been demonstrated that concrete, perceptually rich objects are more likely to be considered objects than symbols denoting other entities (DeLoache, 1987; 2000). In a series of studies by DeLoache and colleagues, very young children were shown the location of a toy in either a photograph or a physical model of a scale room. They were then asked to retrieve the toy from the actual room. Almost all (88%) of the children shown the photograph were able to make an errorless retrieval of the toy, while only 16% of the children shown the physical model were able to do so. When the model was placed behind a screen, children's retrieval rate improved. Furthermore, slightly older children are very successful in this task. However, when older children were encouraged to play with the model, performance dropped significantly. These studies demonstrate that children have difficulty treating perceptually rich objects as symbols. Decreasing the salience of the object increased the ease of its symbolic use.

Second, there is a large body of literature on analogy (analogy is variant of transfer of knowledge from one domain to another) indicating that properties that are not a part of to-be-learned knowledge (i.e., surface features) may hinder rather than facilitate learning (e.g., Ross, 1987; 1989).

Third, there is recent evidence that there might be a competition between abstract and concrete representations of the same situation, and salient concrete representations may distract learners from more abstract regularities (Goldstone & Sakamoto, 2003).

Finally, there is evidence that transfer from abstract instantiations of knowledge may be in fact easier than transfer from concrete to abstract instantiations (Bassok & Holyoak, 1989). Bassok and Holyoak examined transfer between more abstract algebraic knowledge and more concrete physics knowledge, namely between arithmetic-progression problems and isomorphic constant-acceleration problems. High school and college students (who were unfamiliar with both of these domains) learned one of these topics and then were posed word problems involving the other topic. The measure of transfer was whether the learned method had been applied to the structurally isomorphic problems in the unstudied domain. Students who had learned arithmetic-progression first easily and spontaneously applied the learned method to correctly solve constant-acceleration problems. However, the students who learned the physics topic showed essentially no transfer of method to the arithmetic-progression problems. The results of this study suggest that transfer is more likely to occur from a more abstract instantiation to a concrete isomorph.

While the Bassok and Holyoak study (1989) certainly implies that more transfer occurs from abstract to concrete domains, confounds in the study limit such a broad conclusion. The chosen topics in mathematics and physics, as any mathematical and physical topics, do not exist in isolation. Any individual has many associations with each, including related prior learning as well as attitudes and beliefs. Specifically, the amount of mathematics learned through elementary, middle, and high school is significantly more than the amount of physics learned. This disparity of learning most likely exists between mathematics and any of its isomorphic applications. Furthermore, through the course of education, students develop an expectation that mathematical concepts can effectively and appropriately be applied to other domains such as physics, chemistry, economics, to name just a few. It is doubtable that student have as strong expectations that scientific strategies can be used to solve purely mathematical problems.

The purpose of this study was to investigate transfer across two isomorphic domains: one that used a set of abstract symbols, and another that used concrete perceptually-rich objects. To eliminate potential confounds stemming from prior knowledge, both domains were artificially constructed to be algebraic Abelian groups of order three. In other words, each is isomorphic to the integers under addition modulo three. Therefore, both domains included three classes of entities and a set of specific transformation rules described in Figure 1. The first, more abstract, domain (hereafter "Mathematics") was presented to the participants as a symbolic language in which three types of symbols, denoted as ♦, ❄, and ★, combine to yield a resulting symbol. The combination of symbols is expressed as written statements such as ★, ❄ → ♦. The second, more concrete, domain (hereafter "Science") involved interactions between three-dimensional objects from three classes. The objects dynamically interact to form a resulting object. The appearance of the objects and interactions was designed to be dissimilar to any particular science.

The goal of the reported experiment was to investigate transfer across the two isomorphic artificial domains. Transfer was measured by comparing average test scores on a given domain as a function of prior learning of another domain.

# Method

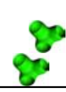
## Participants

Participants in the experiment were undergraduate students from Ohio State University who received partial credit for an introductory psychology course. Thirty students participated in the experiment. Fifteen participants were in the math-then-science condition and fifteen were in the science-then-math condition. Information was presented to individual participants via computer.

## Materials and Design

Materials included two sets of entities (i.e., abstract meaningless symbols and concrete, perceptually-rich objects), and a set of transformations rules (see Table 1).

**Table 1. Example of stimuli and transformation rules across the two domains.**

| | Mathematics | Science | |
|---|---|---|---|
| Elements | 🌢 ❄ ★ |  | |
| Associativity | For any elements *x*, *y*, *z*,: ((*x* , *y*) , *z*) is equivalent to (*x* , (*y* , *z*)) | | |
| Commutativity | For any elements *x*, *y*: *x* , *y* is equivalent to *y* , *x* | | |
| Identity | There is an element, I, such that for any element, *x*: *x* , I is equivalent to *x* | | |
| Inverses | For any element, *x*, there exists another element, *y*, such that: *x*, *y* is equivalent to I | | |
| Specific Rules: | 🌢 is the identity |  is the identity | |
| | | Operands | Result |
| | ❄ , ❄ → ★ |  |  |
| | ★ , ★ → ❄ |  |  |

Information about each domain was given as a computer presentation. The training in both domains was essentially isomorphic. The rules of the domain, namely commutativity, associativity, and the rules governing specific elements, were explicitly stated.

The experiment included four phases presented over one hour: (1) Training in domain X, (2) Test in domain X, (3) Training in domain Y, (4) Test in domain Y, with participants randomly assign to a particular order of learning (i.e., math-then-science or science-then-math).

Training included introduction of transformation rules, followed by questions with feedback. Several detailed examples were given. Testing consisted of twenty multiple choice questions designed to measure recall of the given rules and deeper conceptual understanding of the system. For both domains, the test questions were completely isomorphic and were presented in the same order.

The presentation of the two domains differed by storyline. The artificial mathematics was presented as a symbolic language discovered on an archaeological search. Symbols of different categories combine to yield a resulting symbol. The artificial science was explained to be a phenomenon observed on a planet outside of our solar system. Objects from different classes of shapes interact to form a resulting shape. The presentation of the artificial science included movie clips demonstrating the interactions. Two or more objects move toward each other. When they come in contact, an interaction occurs and results in a predictable object.

Each subject was randomly assigned to one of two orders: math-then-science (M→Sc) or science-then-math (Sc→M). Participants in the first group received training and testing in the artificial mathematics immediately followed by training and testing in the artificial science. Subjects in the second group received training and testing first in the science and then in the mathematics. Following training and testing in both domains, a brief interview was conducted.

Students' scores on the mathematics and science tests were recorded. They were also asked to rate the similarity of the two domains on a scale from one to five. A rating of one indicated that the domains are completely different and a rating of five indicated that the domains are structurally identical with different representations of the objects.

Test scores for mathematics and science were compared across the two conditions, math-then-science and science-then-math. Transfer due to mathematics first was taken to be the difference in the average science score for M→Sc and the average science score for Sc→M. In other words, transfer is the improvement in science score due to having previously learned the mathematics. Similarly, transfer due to science was taken to be the difference in average mathematics score for Sc→M and the average mathematics score for M→Sc.

## Procedure

All training and testing was presented on a computer screen. Participants were tested in a quiet room in a lab by a female experimenter. They proceeded through training and testing at their own pace, and their responses were recorded by the researcher. After training and testing in both domains, a brief interview was conducted.

## Results and Discussion

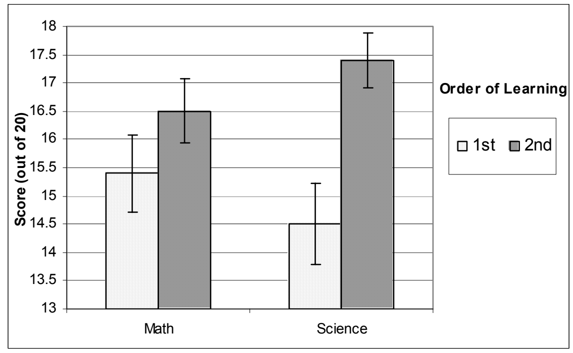
Students were able to learn the artificial mathematics and the artificial science. With the exception of one student, all test scores were significantly above chance (i.e., 7/20) in both math-then-science and science-then-math conditions. The one student who had a science score of 9/20 was removed from the data analysis. This score is not significantly different than chance and is also greater than two standard deviations from the mean (mean = 16.8, standard deviation = 2.8).

All students indicated that they noticed similarities between the two topics. Under both conditions the students rated the domains as highly similar. On a similarity scale from one to five, the mean rating given by math-then-science participants was 4.4 ($SD$ = .65). The mean rating given by science-then-math students was 4.6 ($SD$ = .63).

The data on transfer across the domains are presented in Figure 1. These data were subjected to a 2 (Domain: Math vs. Science) by 2 (Order: Learned First vs. Learned Second) mixed ANOVA with Domain as a repeated measure. The analysis revealed a significant Domain by Order interaction, $F(1, 27) = 24.15$, $p < .0001$. At the same time, none of the main effects was significant, both $p$s > .28.

Planned comparisons indicated that there was a significant difference in performance as a function of learning order. Students in the math-then-science condition performed significantly better on the science test than students in the science-then-math condition, independent-samples $t(24) = 3.26$, $p<.01$. However, there was no significant difference in mathematics scores across conditions, $p > .229$.

**Figure 1. Mean test scores for mathematics and science shown as first and second domain studied.**




The higher average science score for the M→ Sc group suggests that their prior knowledge of the mathematics improved their learning of the science. Therefore, significant transfer was found from the abstract symbolic domain to the concrete, but the reverse was not found.

In order to understand why the symbolic representation promoted transfer, while the concrete representation did not, it is necessary to take a closer look at the process of transfer. Not only does transfer require recognition and mapping of analogous relational structure from a source domain to a target domain, the elements of the source domain need to act as symbols. In other words, the objects or aspects of the source domain need to act as placeholders that can refer to something else, namely the objects of the target domain. Concrete representations are perceptually rich and consequently engage the perceptual system. Perceptually rich representations can easily convey associated properties and overall similarity (Goldstone & Barsalou, 1998). However, the specific characteristics of objects or elements are often irrelevant to concepts. Maintaining dissociation between the relational structure and the characteristics of the given elements is often crucial to accurate analogical reasoning. The salience of surface attributes often misleads students in the course of problem solving by distracting them from the underlying structure (Ben-Zeev & Star, 2001; Gholson, Smither, Buhrman, Duncan, & Pierce, 1997; Holyoak, Junn, & Billman, 1984; Holyoak & Koh, 1987; Schoenfeld & Herrmann, 1982). Perceptual objects convey affordances that may be helpful, but may also be irrelevant to the underlying concepts.

However, concrete representations may be beneficial under some conditions. For example Goldstone and colleagues (Goldstone, Son, & Patton, under review) have argued that maximum transfer occurs through "concreteness fading" where concrete representations progressively become idealized.

The conclusions of their study were that transfer is promoted through multiple representations. Furthermore, the authors conclude that while idealized displays promote internal representation not deeply embedded in a single domain, concrete displays have the advantage of a strong intuitive link between the real world and the modeled world. In other words, concepts can get a partial free ride from familiar concrete instances. However, in the course of learning mathematics and science, there are many concepts for which obvious concrete models may not exist. In the absence of a familiar concrete model on which concepts can freeload, does an artificially constructed concrete representation have benefits over a symbolic representation?

This notion of concepts getting a free ride from concrete representations as suggested by Goldstone and his colleagues is certainly appealing and is intuitively very reasonable. From a pedagogical perspective, there seems to be definite merit in concreteness fading provided that instructors do not allow learning to become deeply embedded in the concrete example. For students, the concept may become the concrete model and not the abstraction necessary for true understanding and transfer. Furthermore, many concepts may not have obvious and familiar representation in the real world. Mathematical concepts, by their very nature, are not bound to concrete contexts. Their transfer depends on attending to their relational structure and not salient surface features of a particular instance.

The goal of this experiment was to take a closer look at the merits of abstraction and concreteness for transfer. The familiarity or intuitive link between the concrete model and the concept were intentionally removed. No significant transfer from the concrete to the abstract was found, while significant transfer from the abstract to the concrete was exhibited.

Concrete representations may be difficult to treat as symbols in novel, complex concepts. Perceptually rich representations convey more information than leaner representations. As the degree of richness increases, it likely becomes more difficult to recognize the representation as an object itself as well as a reference to its intended referent. Successful transfer requires the elements of the source domain to be treated as symbols. Concrete representations engage the perceptual system. Rich percepts convey much information, a large portion of which is unrelated to a task in question. When that information correlates with the conceptual structure, learning may be facilitated. However, when the attributes are irrelevant to the concept, learners may not see the relevant analogy. Even when the analogy is perceived, it is difficult for rich percepts to be used as symbols, as demonstrated in this experiment. Participants in both the math-then-science and the science-then-math conditions recognized similarities between the domains. However, only the students who learned the symbolic mathematics prior to the concrete science were able to transfer information

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